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Land Locomotion Laboratory

Components Research and Development Laboratories

Research and Engineering Directorate

ON THE STATISTICAL PROPERTIES OF THE GROUND CONTOUR AND ITS RELATION TO THE STUDY OF LAND LOCOMOTION

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ABSTRACT

The ground plays a fundamental role in the study of the ground-vehicle-driver complex which comprises the subject of Land Locomotion. Through the pioneering efforts of people such as Col. M. G. Bekker, formerly of the Land Locomotion Laboratory of the U. S. Army Ordnance Corp, Detroit Arsenal, great progress has been made concerning the role of ground soil properties and mechanics in determining the tractive ability of a vehicle traveling on the surface. However, until quite recently, very little has been studied concerning the role of the ground surface contour in determining the speeds of vehicles especially on off-road conditions.

The authors [3] have found that for a simplified vehicle traveling on a random ground contour there is a quantitative relation between the ground described by its spectral density function and the optimal wheel-base relative to the criterion of maximizing driver's comfort. This was only an initial study but points the way to the fact that the geometry of the ground surface must be clearly studied to try to characterize it in such a way that quantitative results concerning significant vehicle parameters may be derived.

In this report statistical models of the ground surface contour are considered along with possible forms for the two dimensional power spectral density. The advantages and disadvantages of the models are pointed out. Perturbations in the power spectral density of the surface

are studied to determine the magnitude of their effect on an optimum criterion chosen for vehicle parameter studies. Finally, the power spectral densities from an actual ground surface survey are presented and discussed.

PREFACE

This is the fifth in a series of papers studying the motion of vehicles under random excitation. Prior papers are:

a. Land Locomotion Report No. 48, Behavior of a Linear One Degree of Freedom Vehicle Moving With Constant Velocity on a Stationary Gaussian Random Track.

b. Land Locomotion Report No. 56, On the Behavior of a Linear Two Degree of Freedom Vehicle Moving With Constant Velocity on a Track Whose Contour is a Stationary Random Process.

c. Land Locomotion Report No. 65, On the Statistical Analysis of the Motion of Some Simple Vehicles Moving on a Random Track, which also includes an appendix which defines many terms and derives many basic concepts used in the series.

d. Land Locomotion Report No. 66, On the Statistical Analysis of the Motion of Some Simple Two-Dimensional Linear Vehicles Moving on a Random Track.

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I. Introduction

The remarkable speed range of modern passenger and commercial motor vehicles on a modern highway is due partly to the power available in contemporary power packages, partly to the suspension system, partly to the straightness of the highway, but it is mainly due to the smoothness of the surface upon which they operate. One immediately realizes the importance of the smoothness of the surface when these vehicles are required to operate off the road. Certain wheeled military vehicles have nearly equivalent speed ranges when operating on the same types of smooth pavements. It must be remembered, however, that of necessity, a military vehicle may be required to operate off paved roads. Under off-road conditions, these military vehicles like modern passenger and commercial vehicles have a very limited speed range. This fact is, of course, not strange, since the same considerations are used in the design of the three types of vehicles.

The basic design criteria for all wheeled vehicles in most categories is much the same--good speed and control on the highway together with, in some cases, load carrying capacity. A vehicle designed using such criteria does not operate and cannot be expected to operate at even moderate speeds when the assumption that it will operate on a modern smooth highway is changed to the assumption of operation under off-road conditions.

To secure reasonable off-road speeds of wheeled or other types of vehicles, it is obvious that fundamental studies must be made of the factors which limit speed under these conditions.

The Land Locomotion Laboratory, Detroit Arsenal, has advocated that such studies be made. The material presented in this report is results of such studies conducted by the Midwest Applied Science Corporation of Lafayette, Indiana.

This report in particular is concerned with the statistical description of the ground surface, and possible methods of characterization.

We hasten to add that this is a preliminary study of the problem. Therefore, the results must be considered as tentative and apt to change as more terrain is surveyed and the data is analyzed.

II. The Ground - Introductory Remarks

It is a remarkable fact that although the very earliest form of transportation was land locomotion, the way in which the ground characteristics should enter into the design of vehicles has been the least understood of the three major environments of travel, that is, of land, sea, and air travel.

We shall be primarily concerned here with the ground part of the ground-vehicle driver complex. Any study of the Theory of Land Locomotion must start with the ground for it is the primary and continual input into any vehicle that moves by way of its contact with the ground, whether the contact is through tires, tracks, skis, or any other mode.

The characteristics of the ground can be broken up into two major areas, the physical properties of the ground as a material and the geometrical properties of the ground as a two dimensional surface.

The material properties of the ground are those associated with its elasticity, plasticity and trafficability. Bekker [1] , and his associates have made many interesting and important studies of these ground properties and their relation to the Theory of Land Locomotion.

It is the ground as a geometrical surface that has been the least studied relative to the Theory of Land Locomotion. Furthermore,

the surface apparently has not been analytically characterized in any realistic fashion. This can be thought to be mainly due to its irregular nature and the lack, until relatively recently, of methods of analyzing and characterizing surfaces with such peculiarities or --if you will--randomness in elevation. The methods we refer to are those found within the scope of the Theory of Probability and Random Functions.

Only recently have statistical analyses of road surfaces been considered, in particular the work of Hoboult et al [2] in determining the spectral properties of airport runways. Even here ground vehicles were not the primary consideration!

In the studies over the past two years by Bogdanoff and Kozin of the Midwest Applied Science Corporation [3] [4] under contract with the Land Locomotion Laboratory of the Detroit Arsenal OTAC, U.S.A., ground elevation surveys have been performed along lines on typical plowed ground in central Indiana. Spectral analyses were then performed from these surveys. Within their studies the statistical analysis was performed of simple vehicles traveling on a track whose contour is one member function of a random process having spectral properties approximating those found from the surveys. Full details of these studies are given in the next paper. At the time of the writing of this paper the results of these surveys concerning the field as random surface have not been fully appraised. Therefore, even though the statistical properties are known along particular straight

lines on this field, its general two-dimensional statistical character remains unknown. It is probably safe to say that the question of the statistical character of arbitrary ground surfaces is quite new and yet to be answered.

In this paper we shall consider two aspects of the theory of random ground surfaces, the theoretical and the experimental. Under the theoretical considerations we shall discuss a possible analytical model of the random ground surfaces along with the associated two dimension-spectral density functions. The advantages and shortcomings of using these models will also be discussed. It is certain that approximations to ground surfaces through spectral densities will have to be employed in studying the motion of vehicles traveling over random surfaces. Hence, the magnitude and nature of the errors introduced into the analysis by using such approximations will also be studied.

Under the experimental considerations we shall present and discuss the results of a ground surface survey that was designed specifically to try to characterize the spectral properties along straight lines in any direction for that particular field. This survey will be compared with the theoretical results discussed in this paper in order to determine the feasibility of a spectral characterization of the type we propose.

III. The Ground - Random Models

The model of a random surface that has gained great favor among oceanographers and meteorologists studying the surface of the sea[5] [6] is simply the two dimensional analogue of the Rice random functions [7] that are used as models for the random tracks in the paper to follow. To this end, consider the expression

$$1) \qquad z(x,y) = \sum_{k=-n}^{n} c_k \exp i\,(u_k x + v_k y + \Phi_k) \quad ,$$

where c_k , u_k , v_k are real numbers satisfying

$$2) \qquad \begin{aligned} &c_k = c_{-k} \quad , \quad c_0 = 0 \\ &u_{-k} = -u_k \\ &v_{-k} = -v_k \quad , \quad u_k , v_k \text{ have the dimension } L^{-1} \end{aligned}$$

The Φ_k's are independent random phases, each uniformly distributed on the interval $(0, 2\pi)$ satisfying $\Phi_{-k} = -\Phi_k$. Hence, we see that $z(x,y)$ is real and, indeed, is no more than a linear combination of cosine terms with random phases. Conceptually this fits the picture that one has of a ground surface full of miniature mounds and valleys.

One immediate objection to the form 1), is that as $n \to \infty$, $z(x,y)$ must become a Gaussian Process [6] However, it is strongly suspected that the ground surface in general is not Gaussian. It is known that except for a very small error due to non-linearities

in the hydrodynamic equations [5] the sea surface is Gaussian in character and hence 1) modified to include time variation fits the phenomenon quite well.

But this objection can be dispensed with since our studies of vehicle dynamics have been directed toward analysing the so-called second-order properties. These are properties that are determined by the covariance function. It can easily be shown [8] that when studying second order properties of an arbitrary second order random process, that is a process whose mean square exists, the process may be replaced by a Gaussian process with the same covariance function. Thus no contradiction is created by using 1). We realize fully that 1) may not suffice if we must study subtle properties of the vehicle motion that are based upon considerations of the probability densities themselves and not second moments.

Upon taking the first two moments of $z(x,y)$ as given by 1) we find

$$E\left[z(x,y)\right] = 0$$

$$3) \quad E\left[z(x,y)\, z(x+x', y+y')\right] = \sum_{k=-n}^{n} c_k^2 \exp i(u_k x' + v_k y') = \Gamma(x', y')$$

where $E[\;]$ denotes the expectation or ensemble average of the quantity within the brackets.

In words 3) states that the mean is zero and the covariance function exists, being dependent only upon the differences of the abcissas and ordinates. Therefore, by definition, $z(x,y)$ is second order and at least weakly stationary random process.

Because of the weak stationarity expressed by 3), one may define its Fourier inverse as

4) $$\Gamma(x,y) = \int du \int dv\ \mathscr{L}(u,v) \exp i(ux+vy)$$

(Note: Unless explicitly stated otherwise, all integrals will be over the range $(-\infty, \infty)$)

Hence,

5) $$\mathscr{L}(uv) = \frac{1}{4\pi^2} \int dx \int dy\ \Gamma(x,y) \exp i(xu+yv)$$

The function $\mathscr{L}(u,v)$ is the two-dimensional power spectral density function. That is $\mathscr{L}(u,v)\,\Delta u\ \Delta v$ yields approximately the average amount of power of the random process that is found in the frequency rectangle $(u, u+\Delta u) \times (v, v+\Delta v)$.

We may write explicitly,

6) $$\mathscr{L}(u,v)\ \Delta u\ \Delta v = \sum_{k'} c_{k'}^2 ,$$ where the index k' runs

over those frequency pairs (u_k', v_k') that are in the rectangle $(u, u+\Delta u) \times (v, v+\Delta v)$.

This is merely the extension to two dimensions of the power spectral density function that one encounters when studying one-dimensional records.

A major question may now be asked. The question is, "How does one determine a useful approximation, economically, in both time and expense, of $\mathscr{S}(u,v)$ for an actual ground surface?"

Furthermore, what can be said about $\mathscr{S}(u,v)$ restricted to a given path on the surface?

The reason we feel that these are major questions is due to the fact that the power spectrum of the surface appears to be the most readily available way of characterizing the surface statistically. Whether or not one may be able to determine simpler characteristics of a random surface that can be used significantly for vehicle design purposes seems to be unknown at present. Much research remains to be performed in this area.

The remainder of this paper will be devoted to trying to answer the questions posed above.

IV. Spectral Properties Along Paths

We shall first consider straight line paths through arbitrary points (x_0, y_0) having directions determined by the angles formed with the positive x -axis, as shown in Figure 1.

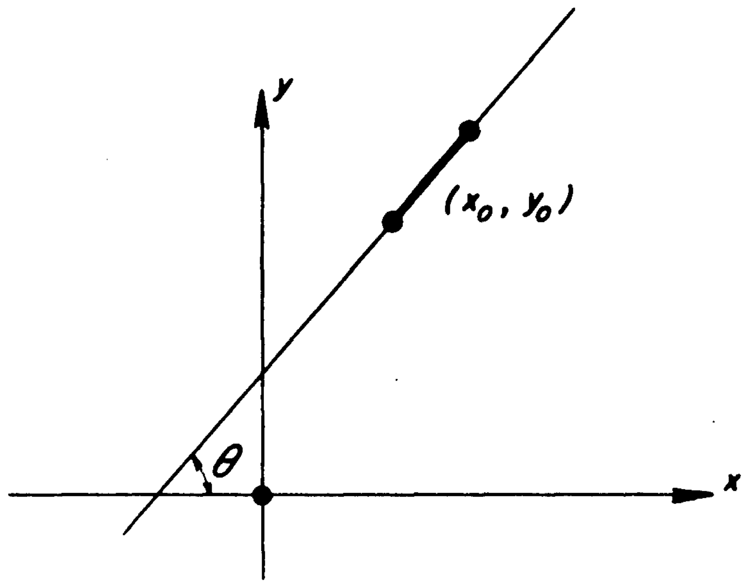

Figure 1.

Following the procedure in [6] consider the transformation

$$
7) \begin{cases}
T; & \xi = (x - x_0)\cos\theta + (y - y_0)\sin\theta \\
 & \eta = -(x - x_0)\sin\theta + (y - y_0)\cos\theta, \\
\text{and its inverse} & \\
T^{-1}; & x = x_0 + \xi\cos\theta - \eta\sin\theta \\
 & y = y_0 + \xi\sin\theta + \eta\cos\theta
\end{cases}
$$

The line in question is given by η identically equal to zero.

Let us first notice that for a weakly stationary process along a line, l, we have

$$8) \qquad \Gamma(\vec{V}') = E\left[z(\vec{V})\, z(\vec{V}+\vec{V}')\right]$$

where $\quad \vec{V} = \vec{V}_0 + k\,\vec{U}$ and $|\vec{U}| = 1$, $\vec{V}_0 \cdot \vec{U} = 0$.

This is given schematically in Figure 2.

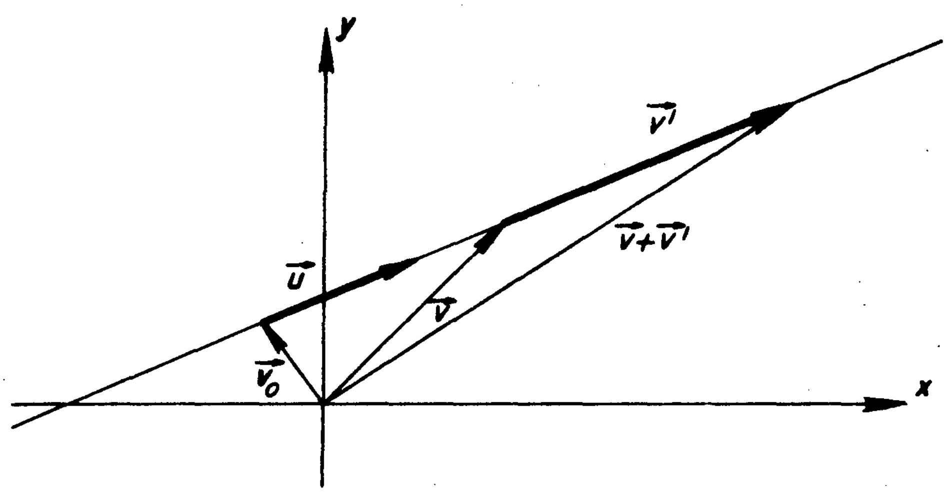

Figure 2.

Hence $\qquad 9) \qquad \vec{V}' = k'\,\vec{U}$

Thus we see that the covariance along any line depends only upon the angle θ, or the direction of the line. Therefore all classes of parallel lines have the same covariance function, and it follows that they shall have the same spectral density function.

Now, using (4) and (7) with $(x_0, y_0)=(0,0)$, we find

$$
10) \left\{
\begin{aligned}
\Gamma'(\xi,\eta) &= \int du \int dv \left\{ \mathcal{E}(u,v) \exp i\left[(\xi\cos\theta - \eta\sin\theta)u + (\xi\sin\theta + \eta\cos\theta)v \right]\right\} \\
&= \int du \int dv \left\{ \mathcal{E}(u,v) \exp i \left[\xi(u\cos\theta + v\sin\theta) + \eta(-u\sin\theta + v\cos\theta) \right]\right\} \\
&= \int du' \int dv' \, \mathcal{E}'(u',v') \exp i \, (\xi u' + \eta v') \,, \quad \text{where} \\
& u' = u\cos\theta + v\sin\theta \\
& v' = -u\sin\theta + v\cos\theta \\
& \mathcal{E}'(u',v') = \mathcal{E}(u(u',v'), v(u',v')), \text{ and the}
\end{aligned}
\right.
$$

Jacobian of the transformation is unity.

Placing $\eta = 0$ for the line in question yields

$$
11) \qquad \Gamma_\theta(\xi) = \Gamma'(\xi,0) = \int du' \int dv' \, \mathcal{E}'(u',v') \exp i \, \xi u' .
$$

But the covariance and spectral density functions are Fourier transform pairs, so that,

$$
12) \qquad \mathcal{E}_\theta(u') = \int dv' \, \mathcal{E}'(u',v') = \int dv' \, \mathcal{E}(u,v) \,, \quad \text{which is}
$$

the spectral density function along the line in the given direction.

At this point we wish to strongly stress the fact that all of **the results so far obtained in this section are based only upon the weak stationarity of the random surface process and not upon explicit forms or processes.**

From 11), we obtain an interesting result concerning weakly stationary random surfaces.

$$\sigma_\theta^2 = \Gamma_\theta(0) = \int du' \int dv' \, g'(u', v') \tag{13}$$
$$= \int du \int dv \, g(u, v)$$

But the last double integral is independent of θ. Therefore, the variance must be the same along any direction! Apparently this has been found to be the case for sea surfaces, but it is not clear that one shall find this to be true for the ground surface. The consideration of weakly stationary random processes to describe the ground surface could break down seriously at this point.

Let us now study paths that are not necessarily straight lines. What can be said about their power spectral density functions? We shall approach the question by first considering the covariance function along a path.

We shall parametrize the path using arc length, s, along the path as the parameter. The elevation for the path $(x(s), y(s))$ on the surface 1) is

$$z(x(s), y(s)) = \sum_{k=-n}^{n} c_k \exp i \left[u_k x(s) + v_k y(s) + \Phi_k \right] \tag{14}$$

The mean is still zero.

For the covariance, one finds,

$$
15) \begin{cases} E\left[z(x(s),\, y(s))\; z(x(s+s'),\; y(s+s'))\right] \\ = E\left[\sum_{k=-n}^{n} \sum_{j=-n}^{n} c_k\, c_j \exp i(u_k\, x(s) + v_k\, y(s) + u_j\, x(s+s') + v_j\, y(s+s') + \Phi_j + \Phi_k)\right] \\ = \sum_{j=-n}^{n} c_j^2 \exp i \left\{u_j \left[x(s+s') - x(s)\right] + v_j \left[y(s+s') - y(s)\right]\right\} \end{cases} ,
$$

since there can only be a contribution to the average if $k = -j$

But in order for this to be a function of s' alone so that a spectral density function exists along the path, it follows that

$$
16) \quad \begin{cases} x(s+s') - x(s) = g(s') \\ y(s+s') - y(s) = k(s') \end{cases}
$$

for any s, s' along the path.

The obvious example of 16) is the straight line path that we have already considered. Are there any other cases? The answer is easily seen to be no by simply differentiating the relations 16) with respect to s and setting s equal to zero, yielding

$$
17) \quad \frac{dx}{ds}, \frac{dy}{ds} \equiv constant.
$$

Therefore, a spectral density function cannot be expected to characterize paths other than straight lines on a weakly stationary random surface.

V. Spectral Models for Real Ground Surfaces

From the preceding considerations, we know that spectral densities exist along straight lines on weakly stationary random surfaces. We may assume that fields free from streams, shell holes, or other man made singularities such as irrigation channels, etc. have the required stationary properties, so that spectral densities of the ground elevations along straight line paths can be obtained. Our problem in this section is inverse to that considered in the previous section, namely, how can we determine $\zeta(u,v)$ from spectral properties of straight line paths?

From several samples of surveys taken along actual rough ground, it appears that the power spectral density function has the general form for straight lines as given by the exponential function

$$18) \qquad \frac{\sigma^2}{u_0\sqrt{2\pi}} \exp\left(-\frac{u^2}{2u_0^2}\right) \quad , \quad \textit{where}$$

u_0 is a measure of the spread of frequencies about zero and σ^2 is a measure of the area under the curve, that is, the variance. We hasten to add that 18) is by no means definitive, since the statistical characterization of ground surfaces is still in its infancy. Qualitative results may be based upon the form 18), but quantitative results based upon 18), must contain inaccuracies. The magnitude and effect of errors based upon inaccuracies in the estimate of the power spectral density function will be considered in the following section.

Upon initial considerations, there are two possible ways of extending the function 18) into a two dimensional spectral density function. Each of the extensions represent distinct modes of attack on the problem of obtaining the $\mathcal{J}(u,v)$ function in the easiest possible fashion. The functions extensions are

$$
19) \quad \begin{cases} \mathcal{J}(u,v) = \dfrac{\sigma^2\sqrt{ac-b^2}}{\pi} \exp -\left[au^2+2buv+cv^2\right] \\[2ex] \mathcal{J}_\theta(u) = \dfrac{\sigma^2}{u_0(\theta)\sqrt{2\pi}} \exp - \dfrac{u^2}{2u_\theta^2(\theta)} \end{cases}
$$

The first expression is simply an extension of the one dimensional exponential function to a two dimensional one. The second expression represents the variation in 18) along linear paths defined by θ the angle between the line and the positive x-axis.

To explicitly determine the functions 19) one would need a, b, c in the first case. In the second case the variation of u_0 with θ would be needed. The factor σ^2 is an amplification factor that represents the variance. It must be constant along any line as derived above. Notice that $\mathcal{J}(u,v)$=constant in the first expression of 19) correspond to ellipses in the (u,v)-plane.

To help substantiate the form chosen for the first of 19), we notice that one may define moments for $\mathcal{J}(u,v)$ as [6]

$$
20) \qquad m_{pq} = \int du \int dv \; \mathcal{J}(u,v)\, u^p v^q \; .
$$

This allows us to define moments along straight lines in various directions by making use of the transformation given in 10).

$$21) \qquad m_n(\theta) = \int du' \, \mathcal{g}_\theta(u')\, u'^n$$

$$= \int du' \int dv' \, \mathcal{g}'(u', v')\, u'^n$$

$$= \int du \int dv \, \mathcal{g}(u,v)\, (u \cos\theta + v \sin\theta)^n$$

Notice, in particular, that the second moment is

$$22) \quad m_2(\theta) = m_{20} \cos^2\theta + 2\, m_{11} \cos\theta \sin\theta + m_{02} \sin^2\theta .$$

This function of θ is continuous in its domain of definition and must attain its extreme values. However, determining these values is equivalent to finding the stationary values of the quadratic form

$$23) \qquad m_{20}\, x^2 + 2\, m_{11}\, xy + m_{02}\, y^2$$

subject to the conditions $x^2 + y^2 = 1$

The solution to this classical eigen-value problem is given by

$$24) \qquad \begin{vmatrix} m_{20} - \lambda & m_{11} \\ m_{11} & m_{02} - \lambda \end{vmatrix} = 0$$

We have, then, two distinct directions for which $m_2(\theta)$ is stationary. These directions can be considered to correspond to the directions of the major and minor axes of the elliptical power spectral density distribution.

To determine the elliptical power spectral density function for a given field one would have to determine a, b, c. This can be accomplished by surveying three linear paths along three distinct directions. We shall discuss a survey devoted to this task later.

The second of the forms in 19) can be useful if there is a definitely preferred direction of the variations in contour of a particular ground surface. This corresponds to the so-called long crested property in a wave pattern. Such a state would presumably exist shortly after a field had been plowed, or if there happens to be a prevailing weather and wind direction creating a distinct grain to the surface. Figure 3 represents schematically this effect.

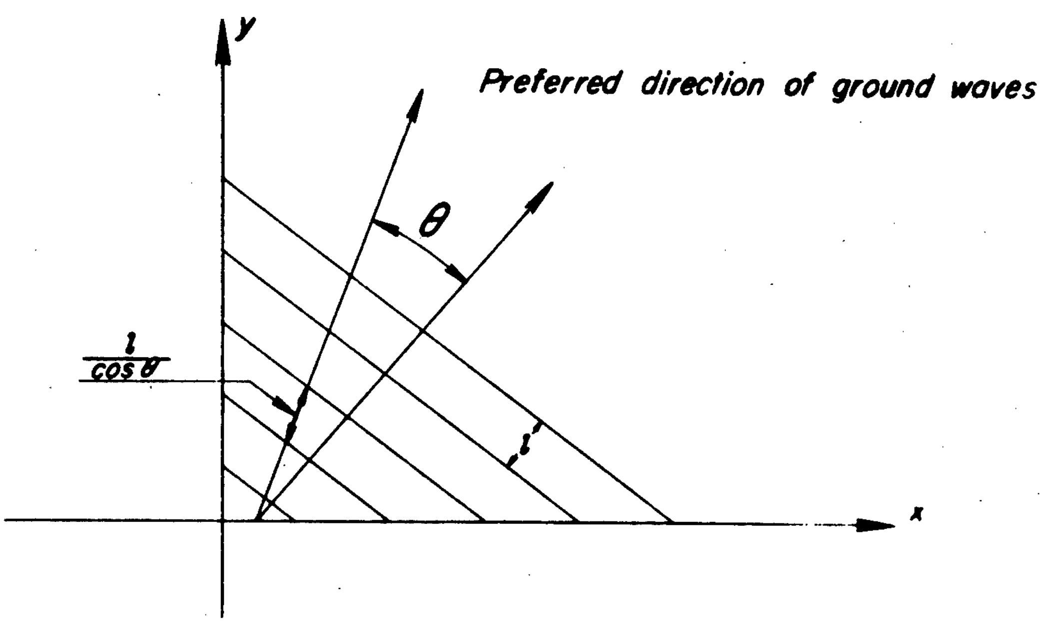

Figure 3.

Therefore for a wave length of l in the preferred direction, we find that the length becomes $\frac{l}{\cos\theta}$ along the direction defined by $\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

The original frequency u becomes $u \cos\theta$ along the new direction. If the power spectral density function along the preferred direction is $\mathcal{E}(u)$ then

$$25) \qquad \mathcal{E}_\theta(u) = \frac{1}{\cos\theta}\, \mathcal{E}\left(\frac{1}{\cos\theta}\right)$$

Hence, using 25) the second of 19) must become

$$26) \qquad \mathcal{E}_\theta(u) = \frac{\sigma^2}{u_0 \cos\theta \sqrt{2\pi}} \exp - \frac{u^2}{2 u_0^2 \cos^2\theta} .$$

Therefore $u_0(\theta)$ becomes $u_0 \cos\theta$. We now have the spectral properties along any linear path on the surface merely from the results found along the preferred direction. For fields that can be recognized to have the properties mentioned above only one survey shall suffice for the determination of its spectral properties.

VI. The Effect of Errors in the Estimate of the Power Spectral Density Function.

The nature of the errors in vehicle parameter studies based upon inaccurate power spectral density forms and measurements is of prime importance in our studies. As one must expect, we may only obtain the spectral density function from surveys within some confidence band.

In the following paper two criteria are used upon which to make optimum judgements concerning the parameters that characterize the simple vehicles considered. The criteria concern the variance of the vertical acceleration and the stationary values of the power spectral density of the vertical acceleration.

One can easily show that the power spectral density of the output of a linear system which is subjected to a stationary random input is simply, assuming one dimensional spectra

$$26) \qquad P(\lambda) = |H(i\lambda)|^2 P(\lambda) = h(\lambda) P(\lambda)$$

where $H(i\lambda)$ is the frequency response of the linear system.

We must recognize that $h(\lambda)$ is really $h(\lambda, \vec{p})$ where $\vec{p}$ is the parameter vector whose values we wish to optimize relative to the criteria mentioned above. We shall consider first order effects on the second of the criteria mentioned above from errors in the power spectral density function.

Consider the power spectral density $P(\lambda) + P^*(\lambda)$, where $P^*(\lambda)$ is

the perturbation or error term. The maximum values of $h(\lambda)\ [P(\lambda)+P^*(\lambda)]$ are determined as usual from the equation

$$27)\qquad \frac{dh(\lambda,p)}{d\lambda}\left[P(\lambda)+P^*(\lambda)\right]+h(\lambda,p)\ \frac{d[P(\lambda)+P^*(\lambda)]}{d\lambda}=0\,,$$

We have specialized the parametric study to only one parameter p. We write the solution to 27) as $\lambda(p)+\lambda^*(p)$ where $\lambda(p)$ is the root if there is no error in the power spectral density estimate. The curve of the maxima as a function of p has the form

$$28)\qquad h(\lambda(p)+\lambda^*(p),p)\left[P(\lambda(p)+\lambda^*(p))+P^*(\lambda(p)+\lambda^*(p))\right]$$

From 27) we may find the first order estimate of $\lambda^*(p)$ by substituting $\lambda(p)+\lambda^*(p)$ into 27), then expanding this expression about $\lambda(p)$. Upon using the fact that

$$29)\qquad \frac{dh}{d\lambda}(\lambda(p),p)\quad P(\lambda(p))+h(\lambda(p),p)\ \frac{dP}{d\lambda}(\lambda(p))=0\,,$$

we find

$$30)\quad \lambda^*(p)=-\ \frac{\frac{d}{d\lambda}\left[h(\lambda(p),p)\ P^*(\lambda(p))\right]}{\frac{d^2}{d\lambda^2}\left[h\ \lambda(p),p)\ P\ \lambda(p))\right]}$$

Proceeding in the same fashion with equation 28), we can determine the first order difference in the curve of the maxima as being

$$31) \qquad h(\lambda(p), p) \; P^*(\lambda(p)).$$

By use of the relations 30) and 31) we can therefore estimate the magnitude of the errors involved by approximating the power spectral density function as exponential.

VII. Ground Surveys and Conclusions

The Midwest Applied Science Corporation has under contract with the Land Locomotion Laboratories undertaken the task of analyzing actual ground survey data to determine the nature of real ground spectra, and make decisions regarding characterizations of the surface. To the best of our knowledge this has not been undertaken in the U. S. A. before. The oceanographers are certainly well ahead in this approach.

Since there is no readily available data for our purpose of studying the ground as a surface, we have performed a survey on a field that has been, in its history, plowed, cultivated and grazed. This field is located in North-central Indiana, adjacent to Purdue University, and is part of the Purdue dairy farms.

The survey on this field was taken along five concentric lines that divide a circle into equal sections, as shown in Figure 4.

The lengths of the lines vary from fifteen hundred feet for line A to twelve hundred feet for line D. The elevations were taken at six inch intervals along each line, thereby yielding a spectral resolution of one cycle per foot.

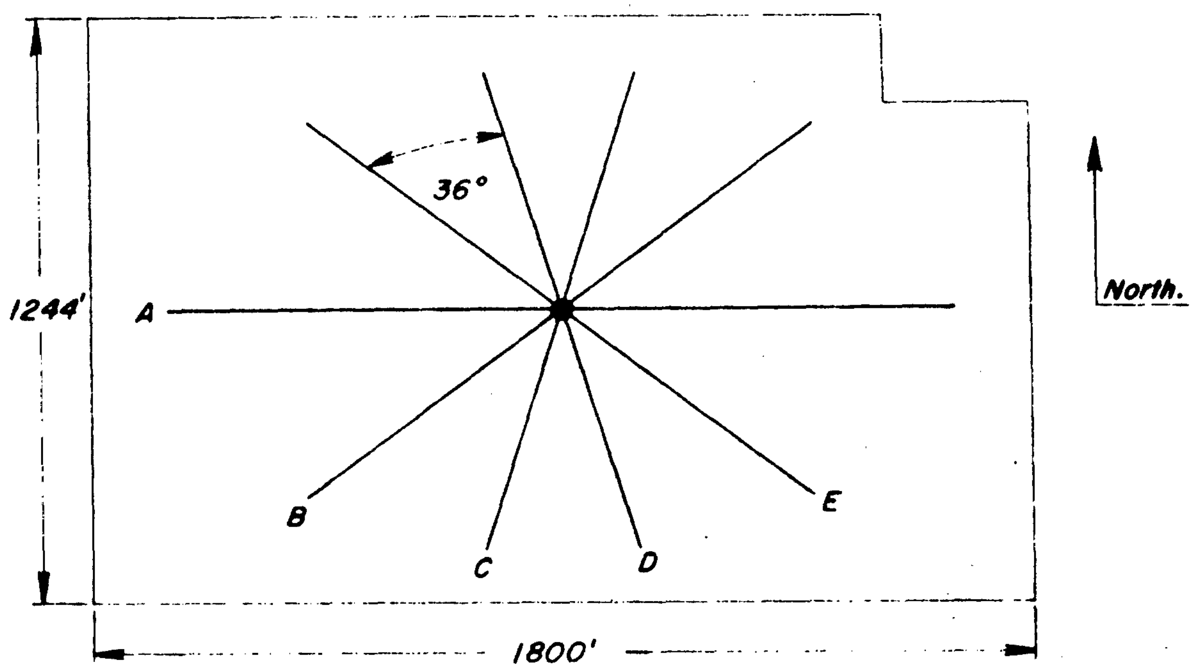

Figure 4.-

It was decided to use the lines in the configuration shown rather than a rectangular grid on the surface due to the fact that we are primarily interested, for the land locomotion problem, in the spectral properties as a function of direction. Hence a polar survey appeared more suitable than a rectangular survey.

The use of five lines was dictated by the fact that if we are to try to fit ellipses to the constant power contours, then this elliptical distribution of the spectral properties with direction would in general be determined by five constants. It is expected that the center of the ellipse will be very near, if not on, the frequency origin. For this reason we

have assumed only three arbitrary constants in the previous analysis. However, we shall continue to use five lines for the first few surface surveys.

To obtain the power spectral densities a numerical analysis was performed on the raw data, and programmed for computation on a digital computer. These are presented along with their ninety per cent confidence limits and their variances in figures 5-8. The numerical procedure used on the raw data was first to subtract out least squares parabolic base lines along each contour in order to delete the extremely low frequencies, for these frequencies do not effect the trafficability of vehicles. The resulting data were then analyzed according to the methods of Blackman and Tukey [9].

The method of deleting the very low frequencies is still open to investigation due to the possibility of introducing high frequency components from the points at which the parabolic base lines are connected. This may account for the large power concentrated around .5 cycles/ft. on line B, and on the large variations of the computed variances. Clearly, the base lines must effect these variances.

The complete analysis of these power spectra is presently in progress, therefore it is too early to say whether this data fits the models described previously. Preliminary studies have indicated a change in the functional form to fit the curve at the lower frequencies more faithfully. But since this is intimately connected with the initial

data reduction procedures we cannot make a decision on this point until further studies are performed.

It is our feeling that the future will see the off-road vehicle that will be adaptively controlled to adjust itself to the terrain that it must operate on. The signal that the control mechanism must operate on is derived from those aspects of the ground surface that will be continuously measured, or sampled, and the optimizing criterion that is built into the system. But what aspects of the ground surface contour should be measured remembering that it is a random surface? We feel that this can be answered by studies along the lines presented in this paper. We hasten to add that this is just an initial study of the general probelm from the off-road mobility point of view. Therefore while particulars may change, the general philosophy presented will remain fixed.

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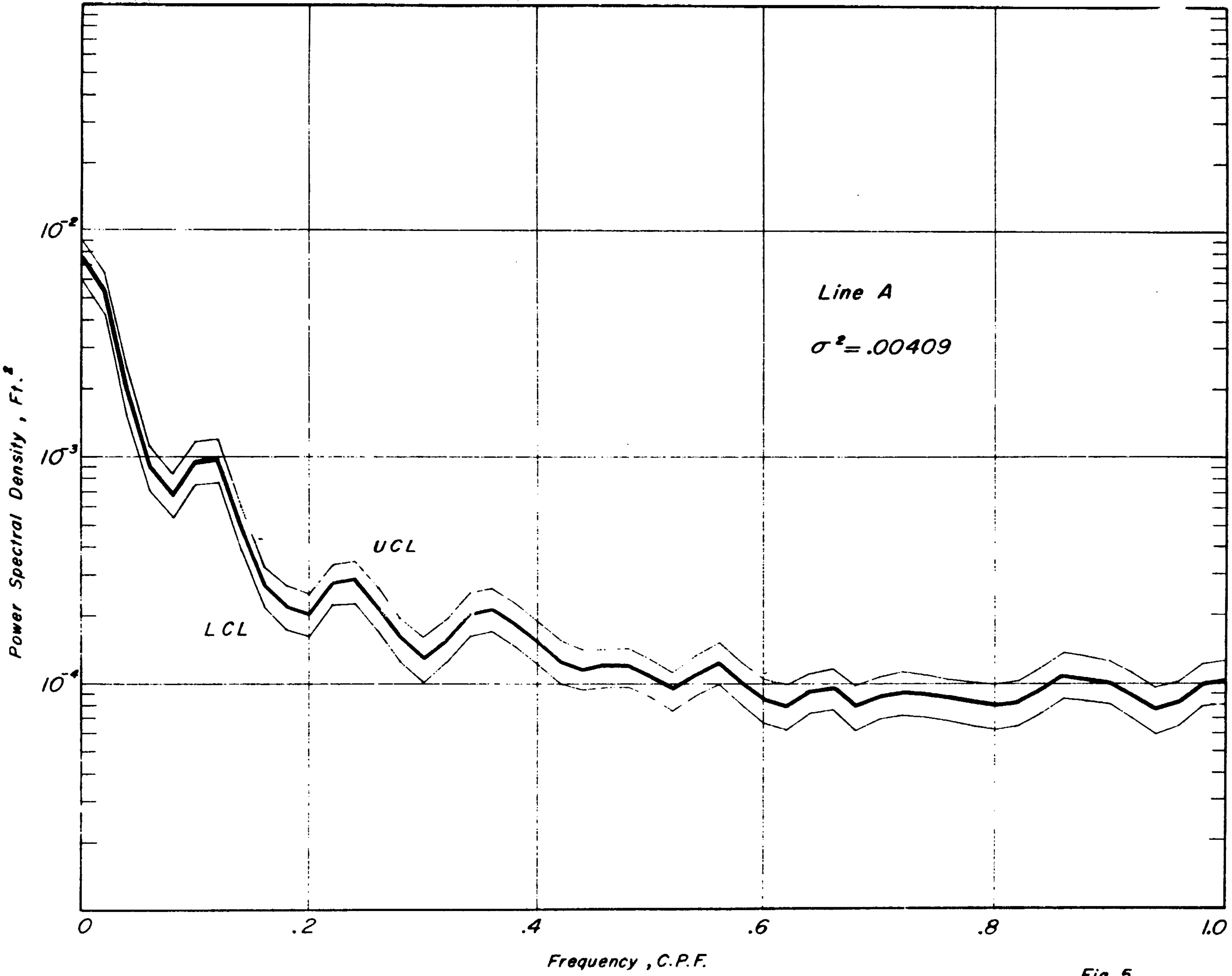
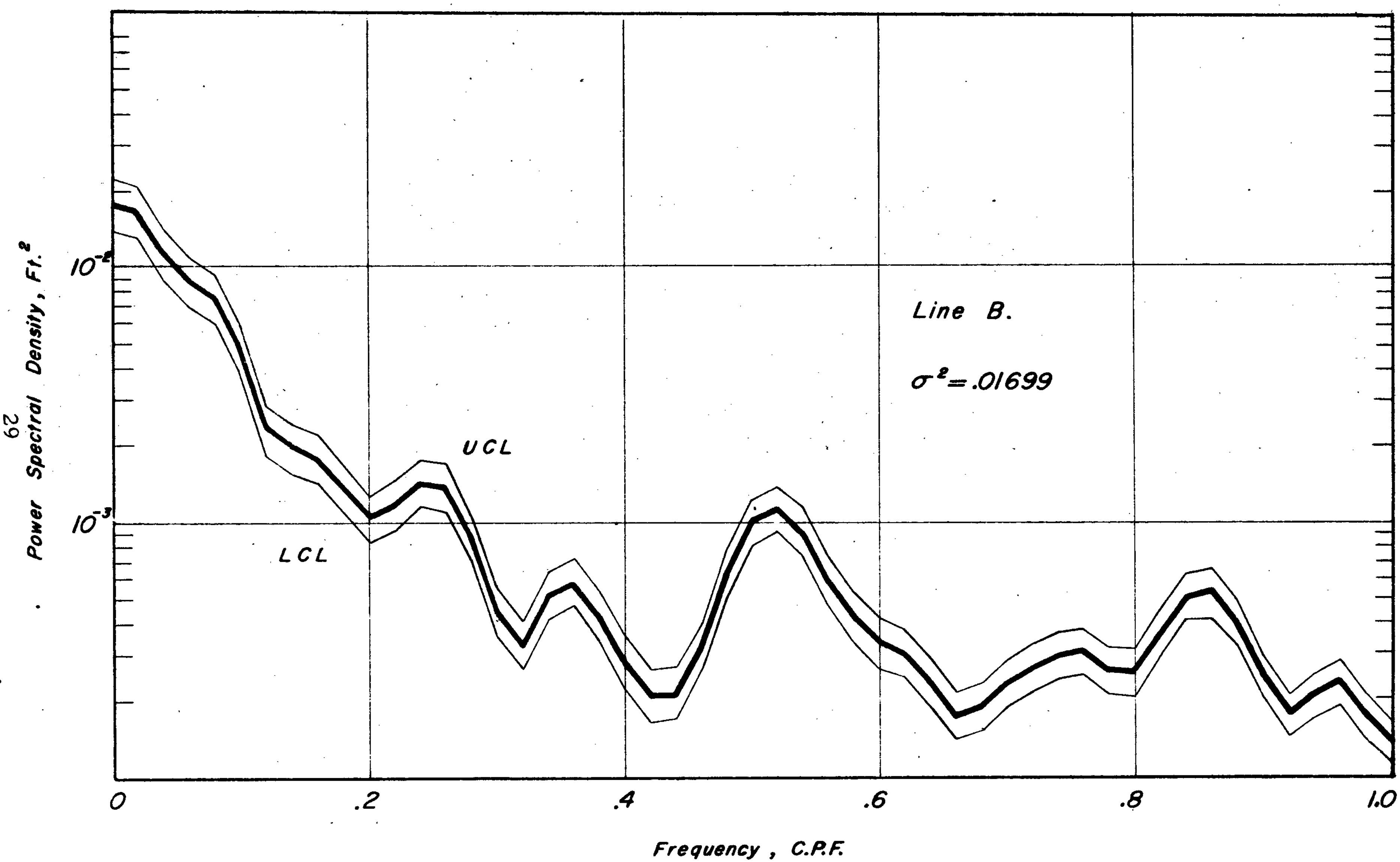


Line A
$\sigma^2 = .00409$
UCL
LCL
10^{-2}
10^{-3}
10^{-4}
Power Spectral Density , Ft.^{2}
0
.2
.4
.6
.8
1.0
Frequency , C.P.F.

Fig. 5



Power Spectral Density, Ft.²
10⁻²
10⁻³
UCL
LCL
Line B.
σ² = .01699
0
.2
.4
.6
.8
1.0
Frequency , C.P.F.

Fig. 6

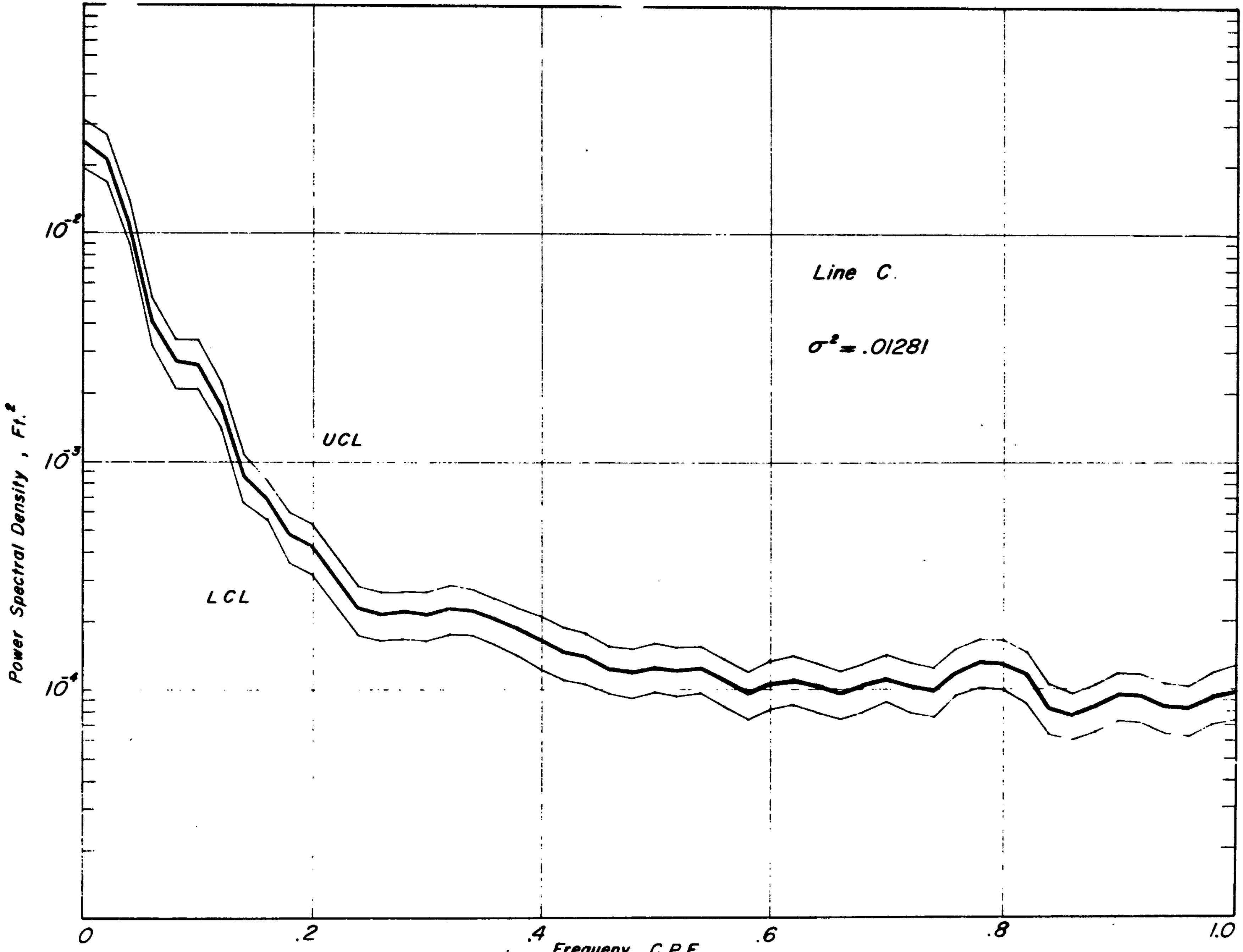
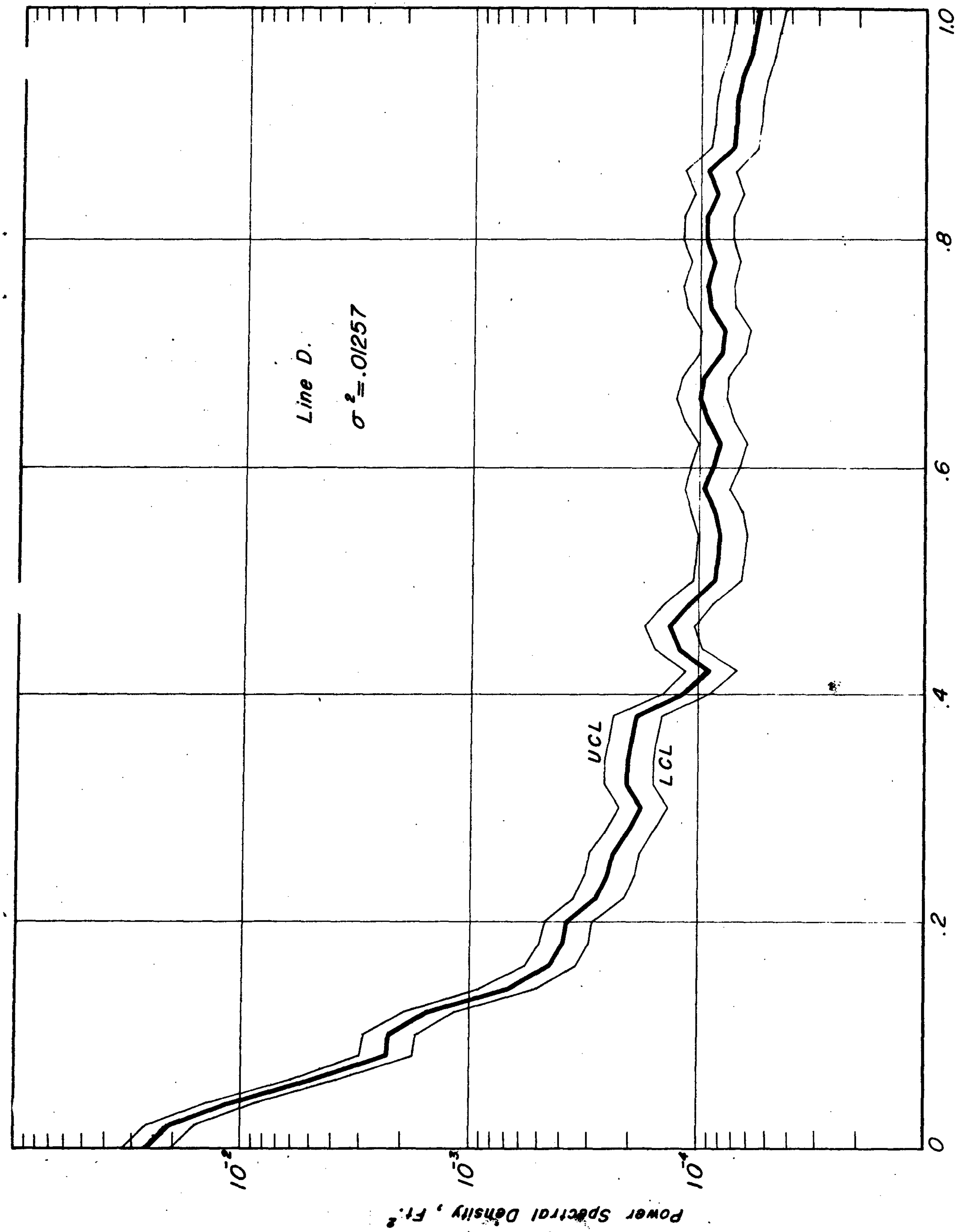
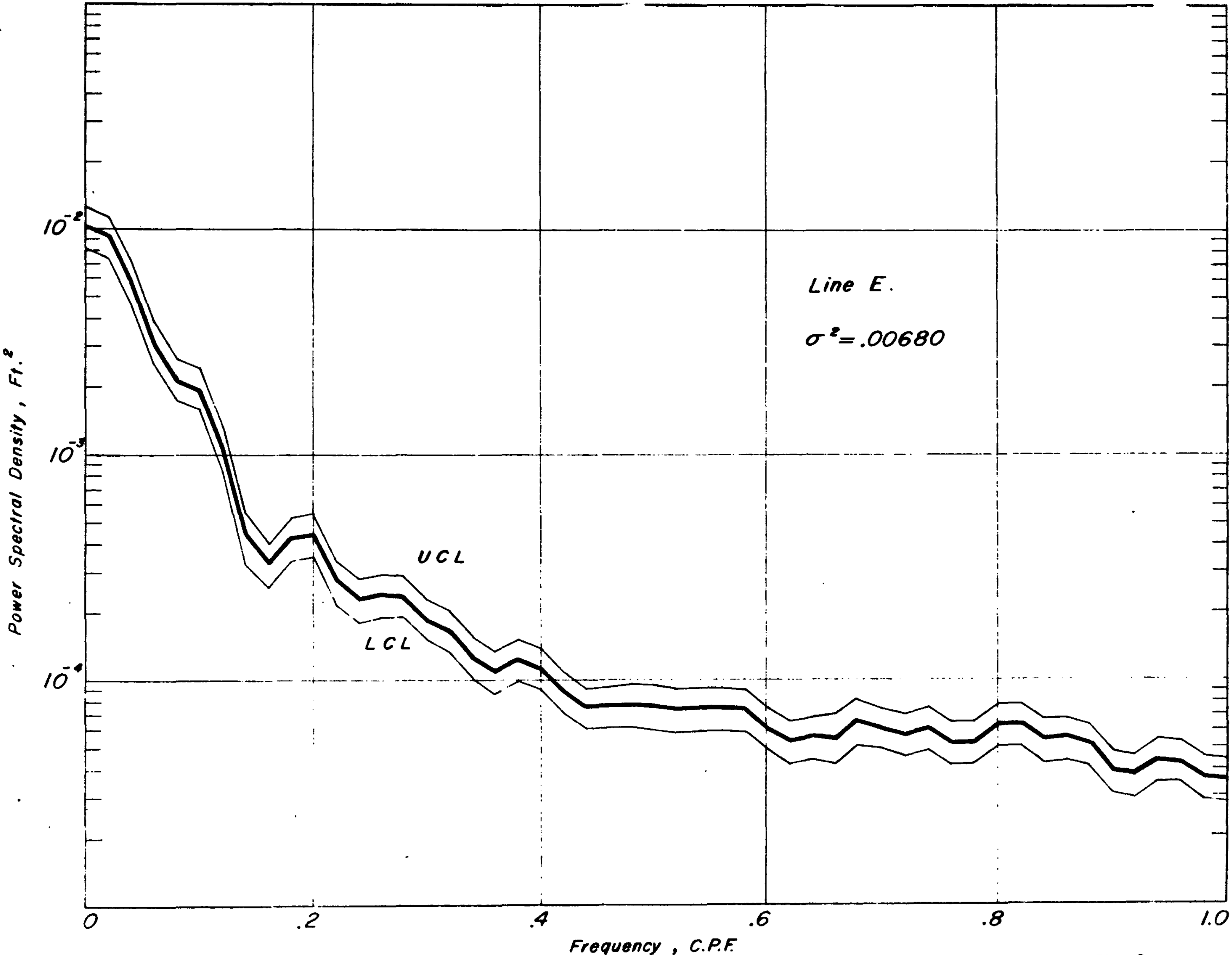

Line C.
σ² = .01281
UCL
LCL
Power Spectral Density , Ft.²
10⁻²
10⁻³
10⁻⁴
0
.2
.4
.6
.8
1.0
Frequeny , C.P.F.

Fig. 7



Line D.
$\sigma^2 = .01257$
UCL
LCL
10^{-2}
10^{-3}
10^{-4}
Power Spectral Density, Ft.²
0
.2
.4
.6
.8
1.0
Frequency, C.P.F.

Fig. 8



Line E.
$\sigma^2 = .00680$
UCL
LCL
10^{-2}
10^{-3}
10^{-4}
Power Spectral Density, Ft.^{2}
0
.2
.4
.6
.8
1.0
Frequency, C.P.F.

Fig. 9

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54 Drag Coefficients of Locomotion over Viscous Soils, Part II

55 Operational Definition of Mechanical Mobility

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ABSTRACT

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1. ORIGINATING ACTIVITY Army Tank Automotive Center Warren, Michigan	**2a. REPORT SECURITY CLASSIFICATION** Unclassified **2b. GROUP** *(For ASTIA use only)*
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4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*	
5. AUTHOR(S) *(Last name, first name, initial)* Bogdanoff, J. L. and Kozin, F.	
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2. Ground contour	8. Ground survey	14.
3. Vehicles	9. Elasticity	15.
4. Statistical analysis	10. Trafficability	16.
5. Ground soil properties	11. Plasticity	17.
6. Spectral density	12. Spectral analysis	18.

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16. BODY OF ABSTRACT (Including if applicable, Purpose; Method or approach; Results; Conclusions, applications and recommendations.)

The purpose of the study was to establish the role of the ground surface contour in determining the speeds of vehicles, especially on off-road conditions.

It was found that for a simplified vehicle traveling on a random ground contour there is a quantitative relation between the ground described by its spectral density function and the optimal wheelbase relative to the criterion of maximizing driver's comfort. Although only an initial study, it points the way to the fact that the geometry of the ground surface must be clearly studied to try to characterize it in such a way that quantitative results concerning significant vehicle parameters may be derived.

Statistical models of the ground surface contour are considered along with possible forms for the two dimensional power spectral density. Perturbations in the power spectral density of the surface are studies to determine the magnitude of their effect on an optimum criterion chosen for vehicle parameter studies. The power spectral densities from an actual ground surface are presented and discussed.

17. INDEXING ANNOTATION

Consideration of statistical models of the ground surface contour, along with possible forms for the two dimensional power spectral density.